Optimal chessboard color change

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The color change allows the folder to make some patterns appear on a paper with one color on each side. One is used for the foreground, the other one, for the background. It is typically a folding sequence related to an "edge effect": Usually, the color cannot change on a model without crossing one edge of the paper, Figure 1a. There are some exceptions as in Figure 1b, but as an edge of the paper is hidden, it cannot be used anymore for color change; consequently, this situation is close to the previous one.

When folding a chessboard from an initial square of paper, the color changes are numerous. There are several chessboards folded from a strip of paper, or using modular origami; much less models use a unique square of paper. It is not surprising that a large square is needed for folding (without cutting) a quite small chessboard. For instance, Max Hulme folded a 8x8 chessboard from a 64x64 square in 1977, [1, pp. 36–38]. More recently, in 1996, Marc Kirschenbaum succeeded in folding it from a 40x40 square of paper [2]. John Montroll produced one in [3] from a 36x36 square of paper. Can a similar chessboard be folded from a smaller square of paper? What is the smallest one?

There are no mathematical proof that a 32x32 paper is the smallest one, but a quite convincing presumption that it is. All is based upon the previous remark about edge effects: there must be an edge part of the paper between each neighbouring chessboard light and dark squares. The pattern of initial edges

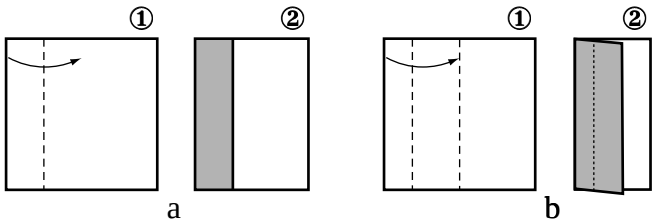


Figure 1: Simple color changes

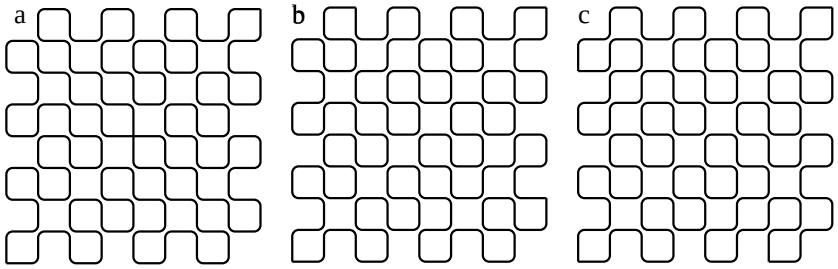


Figure 2: Patterns of initial edges on final model

mapped onto the folded chessboard must fullfill this property. For instance, Figure 2a shows such an edge pattern. Other patterns can be drawn, such as Figures 2b and 2c, but none has more symmetries that the first one. Moreover, each uses a 32x32 initial square of paper.

In order to compare the levels of difficulty, the Figure 3 reports the crease patterns, edge patterns and final models for 2x2, 3x3 and the very elegant 4x4 module of Max Hulme [1, p. 32].

Folding sequence

The chessboard proposed herein has the edge pattern of Figure 2a, and so, is probably "optimal," according to maximal usage of the paper.
Step 1. Pre-crease the 1/16th. Waterbomb base related folds to gather the paper at the center.
Step 2. Rotate to lock. Turn over.
Step 3. Bring edges again toward center.
Step 4. Sink.
Step 5. Book fold. Repeat steps 4-5.
Step 6. Petal fold. Repeat.
Steps 7-8. Fold flaps. Here is a 2x2 pattern.
Step 9. Squash and swivel together.
Step 10. Petal fold (notice the dissymmetry of the fold).
Step 11. Open sink on the left. Reverse fold on the bottom.
Step 12. Open sink again on the left. Same fold as in step 9 on the right.
Step 13. Squash-swivel on the left. Same fold as in step 10 on the right.
Step 14. Same fold as in step 9.
Step 15. Same fold as in step 10.
Step 16. Color change. Repeat steps 8-16 following symmetries.
Step 17. Here is now a 4x4 pattern. Re-fold wider.
Step 18. Color change. Repeat steps 17-18 everywhere needed.
Step 19. Complete.

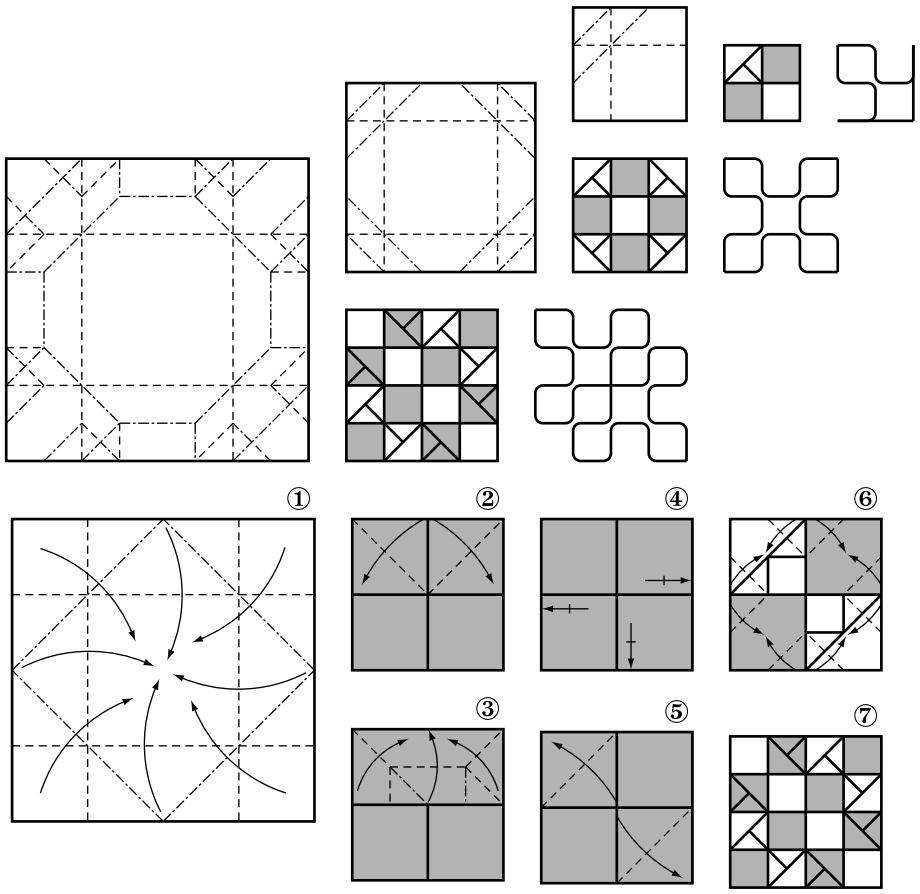
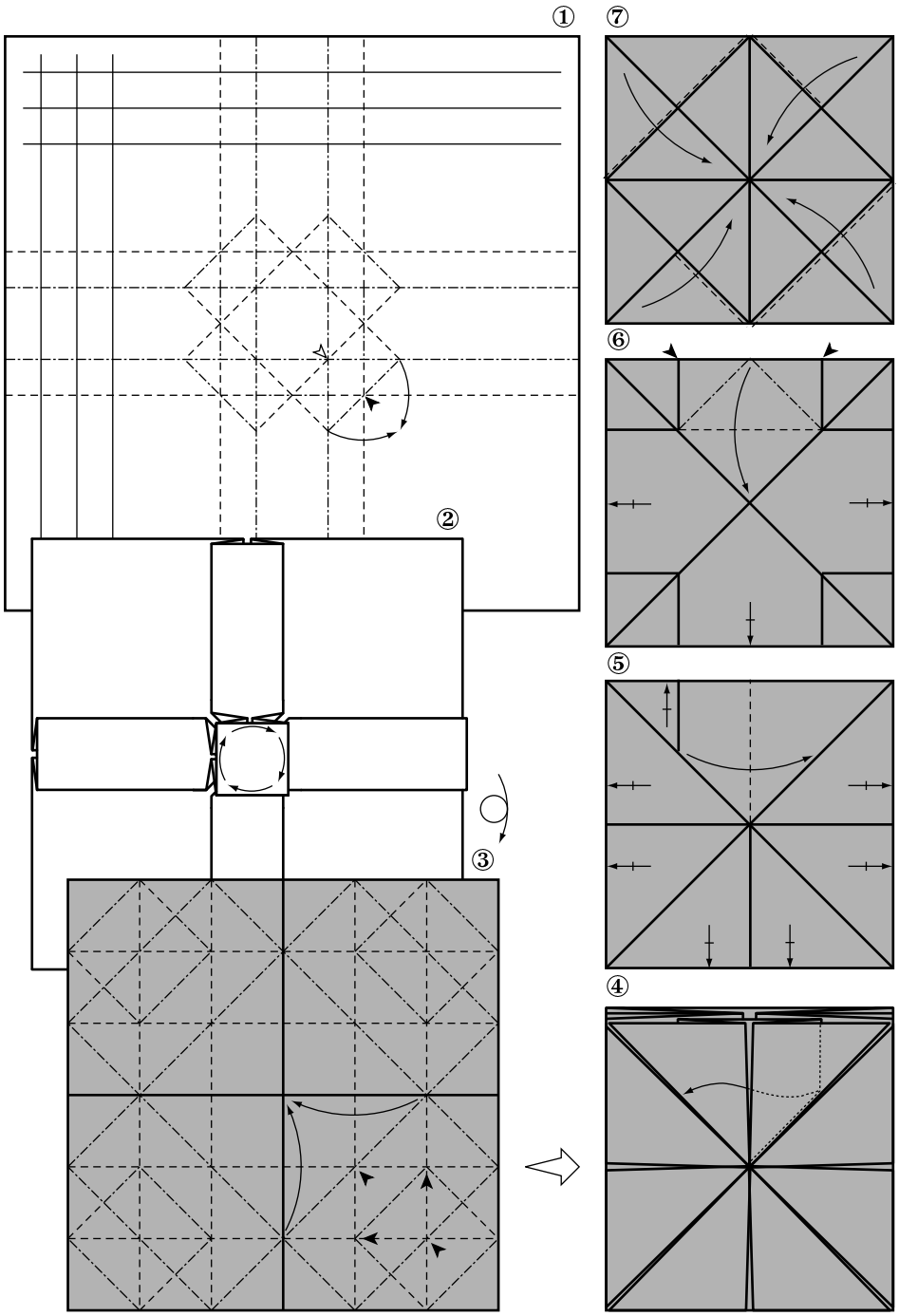


Figure 3: Increasing difficulty of color changes

References

[1] BOS booklet 7, *Chess sets*, 1985.
[2] Marc Kirschenbaum, Chessboard, The Paper, 61, pp. 24–30, 1998.
[3] John Montroll, *Origami Inside-Out*, Dover, 1993.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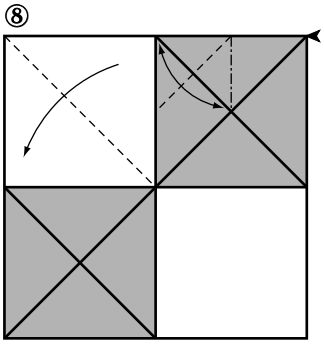

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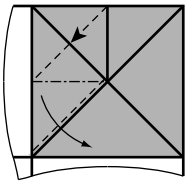

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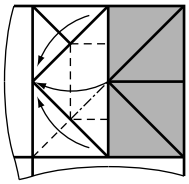

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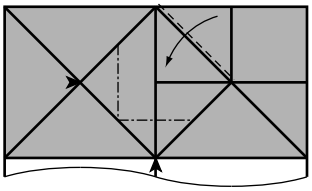

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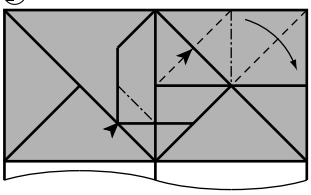

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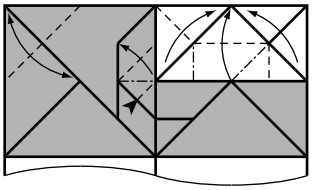

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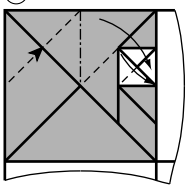

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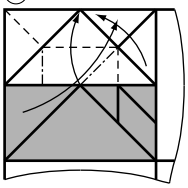

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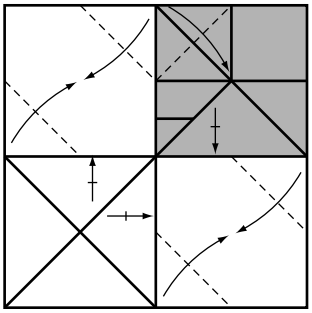

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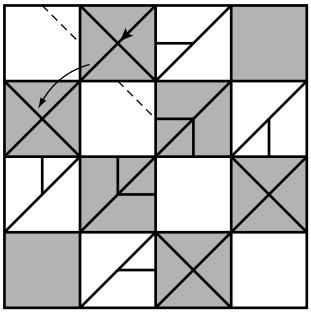

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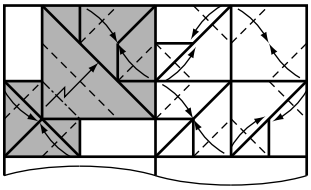

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